WNE Linear Algebra Final Exam

Series A

8 February 2016

Please use separate sheets for different problems. Please provide the following data on each sheet

- **name, surname and your student number,**
- **number of your group,**
- **number of the corresponding problem and its series.**

Problem 1.
Let $V = \text{lin}((1,2,0,1),(0,1,1,-2),(1,3,1,-1),(2,5,1,0))$ be a subspace of $\mathbb{R}^4$.

a) find a system of linear equations which set of solutions is equal to V,
b) let $W_t = \{(x_1,x_2,x_3,x_4) \in \mathbb{R}^4 \mid tx_1 + x_2 - x_3 = 0\}$. For which $t \in \mathbb{R}$ the subspace V is a subset of W_t, i.e. $V \subset W_t$?

Problem 2.
Let $W \subset \mathbb{R}^4$ be a subspace given by the homogeneous system of linear equations

$$\begin{cases} x_1 + x_2 + 2x_3 + x_4 = 0 \\ 3x_1 + 4x_2 + 3x_3 + 2x_4 = 0 \end{cases}$$

a) find a basis and the dimension of the subspace W,
b) find a basis $\mathcal{A}$ of W such that the first two coordinates of the vector (-7,4,1,1) relative to $\mathcal{A}$ are $-1, 1$.

Problem 3.
Let $A = \begin{bmatrix} 4 & 1 \\ -3 & 0 \end{bmatrix}$

a) find matrix $C \in M(2 \times 2; \mathbb{R})$ such that $C^{-1}AC = \begin{bmatrix} 3 & 0 \\ 0 & s \end{bmatrix}$ for some $s \in \mathbb{R}$,
b) compute A^{100}.

Problem 4.
Let $\mathcal{A} = ((1,1,0),(0,0,1),(0,1,2))$ be an ordered basis of $\mathbb{R}^3$ and let $\mathcal{B} = ((2,1),(1,0))$ be an ordered basis of $\mathbb{R}^2$. The linear transformation $\psi \colon \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ is given by the formula $\psi((x_1,x_2,x_3)) = (x_1 + x_2, x_1 - x_2 + x_3)$. The linear transformation $\varphi \colon \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ is given by the matrix $M(\varphi)_{\mathcal{B}}^{\mathcal{B}} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

a) find formula of φ,
b) compute matrix $M(\varphi \circ \psi)_{\mathcal{A}}^{\mathcal{B}}$.

Problem 5.
Let $V = \text{lin}((1,0,1,1),(0,1,1,0),(1,1,2,1))$ be a subspace of $\mathbb{R}^4$.

a) find an orthonormal basis of V,
b) compute the orthogonal projection of $w = (2,0,1,0)$ on $V^{\perp}$.

Problem 6.
Let

$$A = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & -1 & 1 & 0 \\ 1 & 1 & 1 & 2 \\ 2 & 1 & 4 & 5 \end{bmatrix}, \quad B_t = \begin{bmatrix} -t & 2 & -1 & 2 \\ 1 & 1 & 1 & 3 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix},$$

where $t \in \mathbb{R}$.

a) compute $\det A$,
b) for which $t \in \mathbb{R}$ the matrix $A^{-1}B_t$ is invertible?

Problem 7.
Let $Q_t \colon \mathbb{R}^3 \longrightarrow \mathbb{R}$ be a quadratic form given by $Q_t((x_1, x_2, x_3)) = x_1^2 + 5x_2^2 + x_3^2 + 4x_1x_2 + 2tx_2x_3$.

a) for which $t \in \mathbb{R}$ the form Q_t is positive definite?
b) check if Q_t is either positive semidefinite or negative semidefinite for $t = -1$.

Problem 8.
Consider the following linear programming problem $-x_1 + 5x_3 + x_4 + 2x_5 \to \min$ in the standard form with constraints

$$\begin{cases} x_1 + x_2 + 2x_3 + 2x_5 = 5 \\ x_1 + 3x_3 + x_4 + 2x_5 = 3 \end{cases} \text{and } x_i \geqslant 0 \text{ for } i = 1, \ldots, 5$$

a) which of the sets $\mathcal{B}_1 = \{2, 4\}, \mathcal{B}_2 = \{1, 5\}, \mathcal{B}_3 = \{1, 4\}$ are basic? Which of the sets $\mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_3$ are basic feasible and which are basic infeasible?
b) solve the above linear programming problem using simplex method.